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Intuitionistic Fuzzy Generalized Beta Closed Mappings

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ABSTRACT
In this paper we introduce intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings. We investigate some of their properties. We also introduce intuitionistic fuzzy M-generalized beta closed mappings as well as intuitionistic fuzzy M-generalized beta open mappings. We provide the relation between intuitionistic fuzzy M-generalized beta closed mappings and intuitionistic fuzzy generalized beta closed mappings.
Key words and phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings.

I. Introduction

Zadeh [10] introduced the notion of fuzzy sets. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings and study some of their properties. We also introduce intuitionistic fuzzy M-generalized beta closed mappings as well as intuitionistic fuzzy M-generalized beta open mappings. We provide the relation between intuitionistic fuzzy M-generalized beta closed mappings and intuitionistic fuzzy generalized beta closed mappings.

II. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$.
Then
(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.
We shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.
(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
$\text{int}(A) = \cup \{G / G \text{ is an IFOS in X and } G \subseteq A\}$
$\text{cl}(A) = \cap \{K / K \text{ is an IFCS in X and } A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$ [3].

Definition 2.5:[3] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$
(ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$
(iii) *intuitionistic fuzzy α closed set* (IFαCS in short) if $\text{cl}(\text{int}(\text{cl}(A)) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.

Definition 2.6:[3] An IFS A = $\langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be

(i) intuitionistic fuzzy beta closed set (IFβCS for short) if $int(cl(int(A))) \subseteq A$.

(ii) intuitionistic fuzzy beta open set (IFβOS for short) if $A \subseteq cl(int(cl(A)))$.

The family of all IFβCSs (respectively IFβOSs) of an IFTS (X,τ) is denoted by IFβC(X) (respectively IFβO(X)).

Definition 2.7:[3] Let A be an IFS in an IFTS (X, τ). Then the beta interior and the beta closure of A are defined as $\beta int(A) = \cup \{ G / G$ is an IFβOS in X and $G \subseteq A \}$ $\beta cl(A) = \cap \{ K / K$ is an IFβCS in X and $A \subseteq K \}$

Note that for any IFS A in (X, τ), we have $\beta cl(A^c) = (\beta int(A))^c$ and $\beta int(A^c) = (\beta cl(A))^c$ [3]

Definition 2.8:[6] An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy generalized beta closed set (IFGβCS for short) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Every IFCS, IFSCS, IFPCS, IFRCS, IFαCS, IFSPCS and IFβCS is an IFGβCS but the separate converses may not be true in general.[6]

The family of all IFGβCSs of an IFTS (X,τ) is denoted by IFGβC(X).

Definition 2.9:[6] The complement A^c of an IFGβCS A in an IFTS (X,τ) is called an intuitionistic fuzzy generalized beta open set (IFGβOS for short) in X.

Every IFOS, IFSOS, IFPOS, IFROS, IFαOS, IFSPOS and IFβOS is an IFGβOS but the separate converses may not be true in general.[6]

The family of all IFGβOSs of an IFTS (X,τ) is denoted by IFGβO(X).

Definition 2.10:[6] If every IFGβCS in (X, τ) is an IFβCS in (X, τ), then the space can be called as an intuitionistic fuzzy beta $T_{1/2}$ space (IFβ$T_{1/2}$ space for short).

Definition 2.11:[4] A map $f: X \to Y$ is called an intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for each IFCS A in X.

Definition 2.12: [4] A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy open mapping (IFOM for short) if f(A) is an IFOS in Y for each IFOS A in X.

Definition 2.13:[4] A map $f: X \to Y$ is called an

(i) intuitionistic fuzzy semi open mapping (IFSOM for short) if f(A) is an IFSOS in Y for each IFOS A in X.

(ii) intuitionistic fuzzy α open mapping (IFαOM for short) if f(A) is an IFαOS in Y for each IFOS A in X.

(iii) intuitionistic fuzzy preopen mapping (IFPOM for short) if f(A) is an IFPOS in Y for each IFOS A in X.

Definition 2.14: [7] The IFS $c(\alpha , \beta) = \langle x, c_\alpha , c_{1-\beta} \rangle$ where $\alpha \in (0, 1]$ **,** $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP for short) in X.

Note that an IFP $c(\alpha, \beta)$ is said to belong to an IFS A = $\langle x, \mu_A, \nu_A \rangle$ of X denoted by $c(\alpha, \beta) \in A$ if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.15:[8] Let $c(\alpha, \beta)$ be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $c(\alpha, \beta)$ if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Definition 2.16:[6] Let $c(\alpha, \beta)$ be an IFP in (X, τ). An IFS A of X is called an intuitionistic fuzzy beta neighborhood (IFβN for short) of $c(\alpha, \beta)$ if there is an IFβOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Theorem 2.17: Let (X, τ) be an IFTS where X is an IFβ$T_{1/2}$ space. An IFS A is an IFGβOS in X if and only if A is an IFβN of $c(\alpha, \beta)$ for each IFP $c(\alpha, \beta) \in A$.

Proof: Necessity: Let $c(\alpha, \beta) \in A$. Let A be an IFGβOS in X. Since X is an IFβ$T_{1/2}$ space, A is an IFβOS in X. Then clearly A is an IFβN of $c(\alpha, \beta)$.

Sufficiency: Let $c(\alpha, \beta) \in A$. Since A is an IFβN of $c(\alpha, \beta)$, there is an IFβOS B in X such that $c(\alpha, \beta) \in B \subseteq A$. Now $A = \cup \{ c(\alpha, \beta) / c(\alpha, \beta) \in A\} \subseteq \cup \{B_{c(\alpha, \beta)} / c(\alpha, \beta) \in A\} \subseteq A$. This implies $A = \cup \{B_{c(\alpha, \beta)} / c(\alpha, \beta) \in A\}$. Since each B is an IFβOS, A is an IFβOS and hence an IFGβOS in X.

Theorem 2.18: For any IFS A in an IFTS (X, τ) where X is an IFβ$T_{1/2}$ space, $A \in$ IFGβO(X) if and only if for every IFP $c(\alpha , \beta) \in A$, there exists an IFGβOS B in X such that $c(\alpha , \beta) \in B \subseteq A$.

Proof: Necessity: If $A \in$ IFGβO(X), then we can take B = A so that $c(\alpha , \beta) \in B \subseteq A$ for every IFP $c(\alpha , \beta) \in A$.

Sufficiency: Let A be an IFS in X and assume that there exists $B \in$ IFGβO(X) such that $c(\alpha, \beta) \in B \subseteq A$. Since X is an IFβ$T_{1/2}$ space, B is an IFβOS of X. Then $A = \cup_{c(\alpha, \beta) \in A}\{ c(\alpha , \beta) \} \subseteq \cup_{c(\alpha, \beta)\in A} B \subseteq A$. Therefore $A = \cup_{c(\alpha, \beta)\in A} B$ is an IFβOS [6] and hence A is an IFGβOS in X. Thus A $\in$ IFGβO(X).

III. Intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings

In this section we introduce intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings. We study some of their properties.

Definition 3.1: A map f: X $\to$ Y is called an intuitionistic fuzzy generalized beta closed mapping (IFGβCM for short) if f(A) is an IFGβCS in Y for each IFCS A in X.

For the sake of simplicity, we shall use the notation A= $\langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of A = $\langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following examples.
Similarly we shall use the notation B= $\langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of B = $\langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = $\langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, G_2 = $\langle y, (0.3_u, 0.4_v), (0.7_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y, σ) by f (a) = u and f (b) = v. Then f is an IFGβCM.

Theorem 3.3: Every IFCM is an IFGβCM but not conversely.

Proof: Let f: X $\to$ Y be an IFCM. Let A be an IFCS in X. Then f (A) is an IFCS in Y. Since every IFCS is an IFGβCS, f (A) is an IFGβCS in Y. Hence f is an IFGβCM.

Example 3.4: In Example 3.2 f is an IFGβCM but not an IFCM, since G_1^c = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ is an IFCS in X, but $f(G_1^c)$ = $\langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is not an IFCS in Y, since $cl(f(G_1^c))$ = $G_2^c \neq f(G_1^c)$

Theorem 3.5: Every IFαCM is an IFGβCM but not conversely.

Proof: Let f: X $\to$ Y be an IFαCM. Let A be an IFCS in X. Then f(A) is an IFαCS in Y. Since every IFαCS is an IFGβCS , f(A) is an IFGβCS in Y. Hence f is an IFGβCM.

Example 3.6: Let X = {a, b}, Y = {u, v} and G_1 = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, G_2 = $\langle y, (0.8_u, 0.7_v), (0.2_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGβCM but not an IFαCM. Since G_1^c is an IFCS in X but $f(G_1^c)$ = $\langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ is not an IFαCS in Y, since $cl(int(cl(f(G_1^c)))) = 1_\sim \not\subseteq f(G_1^c)$.

Theorem 3.7: Every IFSCM is an IFGβCM but not conversely.

Proof: Let f: X $\to$ Y be an IFSCM. Let A be an IFCS in X. Then f(A) is an IFSCS in Y. Since every IFSCS is an IFGβCS , f(A) is an IFGβCS in Y. Hence f is an IFGβCM.

Example 3.8: In Example 3.6, f is an IFGβCM but not an IFSCM, since Since G_1^c is an IFCS in X but $f(G_1^c)$ = $\langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ is not an IFSCS in Y, since $int(cl(f(G_1^c)))$ = $1_\sim \not\subseteq f(G_1^c)$.

Theorem 3.9: Every IFPCM is an IFGβCM but not conversely.

Proof: Let f: X $\to$ Y be an IFPCM. Let A be an IFCS in X. Then f(A) is an IFPCS in Y. Since every IFPCS is an IFGβCS , f(A) is an IFGβCS in Y. Hence f is an IFGβCM.

Example 3.10: Let X = {a, b}, Y = {u, v} and G_1 = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, G_2 = $\langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$}. Then $\tau = \{0_\sim, G_1, 1_\sim\}$and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y, σ) by f(a) = u and f (b) = v. Then f is an IFGβCM but not an
IFPCM, since $f(G_1^c)$ is an IFCS in Y but not an IFPCS in Y, since $cl(int(f(G_1^c))) \subseteq G_2^c \not\subseteq f(G_1^c)$.

Definition 3.11: A mapping f : X $\to$ Y is said to be an intuitionistic fuzzy M-generalized beta closed mapping (IFMGβCM, for short) if f(A) is an IFGβCS in Y for every IFGβCS A in X.

Example 3.12: Let X = {a, b}, Y = {u, v} and G_1 = $\langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, G_2 = $\langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$.Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y, σ) by f (a) = u and f (b) = v. Then f is an IFMGβCM.

Theorem 3.13 : Every IFMGβCM is an IFGβCM but not conversely.

Proof: Let f: X $\to$ Y be an IFMGβCM. Let A be an IFCS in X. Then A is an IFGβCS in X. By hypothesis f (A) is an IFGβCS in Y. Therefore f is an IFGβCM.

Example 3.14: Let X = {a, b}, Y = {u, v} and G_1 = $\langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, G_2 = $\langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$ and G_3= $\langle y, (0.7_u, 08_v), (0.3_u, 0.2_v)\rangle$. Then

$\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y,\sigma)$ by f (a) = u and f (b) = v. Then f is an IFGβCM but not an IFMGβCM. Since $A = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is IFGβCS in X but $f(A) = \langle y, (0.6_u, 0.8_v), (0.4_u, 0.2_v)\rangle$ is not an IFGβCS in Y, since $f(A) \subseteq G_3$ but $spcl(f(A)) = 1_\sim \not\subset G_3$.

The relation between various types of intuitionistic fuzzy closedness is given in the following diagram.

$$\begin{tikzcd}
 & \text{IF}\alpha\text{CM} \arrow[dr] & & \\
\text{IFCM} \arrow[ur] \arrow[r] \arrow[dr] & \text{IFPCM} \arrow[r] & \text{IFG}\beta\text{CM} & \text{IFMG}\beta\text{CM} \arrow[l] \\
 & \text{IFSCM} \arrow[ur] & &
\end{tikzcd}$$

The reverse implications are not true in general in the above diagram.

Theorem 3.15: Let f: X →Y be a mapping. Then the following are equivalent if Y is an $IF\beta T_{1/2}$ space:

(i) f is an IFGβCM (ii) $\beta cl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.

Proof: (i) ⇒ (ii) Let A be an IFS in X. Then cl(A) is an IFCS in X. (i) implies that f(cl(A)) is an IFGβCS in Y. Since Y is an $IF\beta T_{1/2}$ space, f(cl(A)) is an IFβCS in Y. Therefore $\beta cl(f(cl(A))) = f(cl(A))$. Now $\beta cl(f(A)) \subseteq \beta cl(f(cl(A))) = f(cl(A))$. Hence $\beta cl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.

(ii)⇒(i) Let A be any IFCS in X. Then cl(A) = A. (ii) implies that $\beta cl(f(A)) \subseteq f(cl(A)) = f(A)$. But $f(A) \subseteq \beta cl(f(A))$. Therefore $\beta cl(f(A)) = f(A)$. This implies f(A) is an IFβCS in Y. Since every IFβCS is an IFGβCS, f(A) is an IFGβCS in Y. Hence f is an IFGβCM.

Theorem 3.16: Let f: X →Y be a bijection. Then the following are equivalent if Y is an $IF\beta T_{1/2}$ space:
(i) f is an IFGβCM (ii) $\beta cl(f(A)) \subseteq f(cl(A))$ for each IFS A of X (iii) $f^{-1}(\beta cl(B)) \subseteq cl(f^{-1}(B))$ for every IFS B of Y.

Proof: (i) ⇔ (ii) is obvious from Theorem 3.15.
(ii) ⇒ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. Since f is onto , $\beta cl(B) = \beta cl(f(f^{-1}(B)))$ and (ii) implies $\beta cl(f(f^{-1}(B))) \subseteq f(cl(f^{-1}(B)))$. Therefore $\beta cl(B) \subseteq f(cl(f^{-1}(B)))$. Now $f^{-1}(\beta cl(B)) \subseteq f^{-1}(f(cl(f^{-1}(B)))$. Since f is one to one, $f^{-1}(\beta cl(B)) \subseteq cl(f^{-1}(B))$.
(iii) ⇒ (ii) Let A be any IFS of X. Then f(A) is an IFS of Y. Since f is one to one, (iii) implies that $f^{-1}(\beta cl(f(A)) \subseteq cl(f^{-1}f(A)) = cl(A)$. Therefore $f(f^{-1}(\beta cl(f(A)))) \subseteq f(cl(A))$. Since f is onto $\beta cl(f(A)) = f(f^{-1}(\beta cl(f(A)))) \subseteq f(cl(A))$.

Theorem 3.17: A mapping f: X → Y is an IFGβCM if and only if for every IFS B of Y and for every IFOS U containing $f^{-1}(B)$, there is an IFGβOS A of Y such that $B \subset A$ and $f^{-1}(A) \subset U$.
Proof: Necessity: Let B be any IFS in Y. Let U be an IFOS in X such that $f^{-1}(B) \subset U$, then U^c is an IFCS in X. By hypothesis $f(U^c)$ is an IFGβCS in Y. Let $A = (f(U^c))^c$, then A is an IFGβOS in Y and $B \subset A$. Now $f^{-1}(A) = f^{-1}(f(U^c))^c = (f^{-1}(f(U^c)))^c \subset U$.
Sufficiency: Let A be any IFCS in X, then A^c is an IFOS in X and $f^{-1}(f(A^c))^c \subset A^c$. By hypothesis there exists an IFGβOS B in Y such that $f(A^c) \subset B$ and $f^{-1}(B) \subset A^c$. therefore $A \subset (f^{-1}(B))^c$. Hence $B^c \subset f(A) \subset f(f^{-1}(B))^c \subset B^c$. This implies that $f(A) = B^c$. Since B^c is an IFGβCS in Y, f(A) is an IFGβCS in Y. Hence f is an IFGβCM.

Theorem 3.18: Let f: X → Y be a bijective map where Y is an $IF\beta T_{1/2}$ space. Then the following are equivalent:
(i) f is an IFGβCM
(ii) f(B) is an IFGβOS in Y for every IFOS B in X.
(iii) $f(int(B)) \subseteq cl(int(cl(f (B))))$ for every IFS B in X.
Proof: (i) ⇒ (ii) is obvious.
(ii) ⇒ (iii) Let B be an IFS in X, then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFGβOS in Y. Since Y is an $IF\beta T_{1/2}$ space, f(int(B)) is an IFβOS in Y. Therefore $f(int(B)) = \beta int(f(int(B))) = f(int(B)) \cap cl(int(cl(f(int(B))))) \subseteq cl(int(cl(f(int(B))))) \subseteq cl(int(cl(f(B))))$.
(iii) ⇒ (i) let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, $f(int(A^c)) = f(A^c) \subseteq cl(int(cl(f(A^c)))))$. That is $int(cl(int(f(A)))) \subseteq f(A)$. This implies f(A) is an IFβCS in Y and hence an IFGβCS in Y. Therefore f is an IFGβCM.

Theorem 3.19: Let f: X → Y be a bijective map where Y is an $IF\beta T_{1/2}$ space. Then the following are equivalent:
(i) f is an IFGβCM
(ii) f(B) is an IFGβCS in Y for every IFCS B in X.
(iii) $int(cl(int(f (B)))) \subseteq f(cl(B))$ for every IFS B in X.
Proof: (i) ⇒ (ii) is obvious.
(ii) ⇒ (iii) Let B be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFGβCS in Y. Since Y is an $IF\beta T_{1/2}$ space, f(cl(B)) is an IFβCS in Y. Therefore $f(cl(B)) = \beta cl(f(cl(B))) = f(cl(B)) \cup int(cl(int(f(cl(B))))) \supseteq int(cl(int(f(cl(B))))) \supseteq int(cl(int(f(B))))$.
(iii) ⇒ (i) let A be an IFCS in X. By hypothesis,

f(cl(A)) = f(A) $\subseteq$ int(cl(int(f(A)))). This implies f(A) is an IFβCS in Y and hence an IFGβCS in Y. Therefore f is an IFGβCM.

Definition 3.20: A mapping f : X → Y is said to be an intuitionistic fuzzy generalized beta open mapping (IFGβOM for short) if f(A) is an IFGβOS in Y for each IFOS in X.

Theorem 3.21: If f : X → Y is a mapping. Then the following are equivalent if Y is an IFGβ$T_{1/2}$ space:
(i) f is an IFGβOM.
(ii) f(int(A)) $\subseteq$ βint(f(A)) for each IFS A of X.
(iii) int(f^{-1}(B)) $\subseteq$ f^{-1}(βint(B)) for every IFS B of Y.

Proof: (i) ⇒ (ii) Let f be an IFGβOM. Let A be any IFS in X. Then int(A) is an IFOS in X. (i) implies that f(int(A)) is an IFGβOS in Y. Since Y is an IFβ$T_{1/2}$ space, f(intA)) is an IFβOS in Y. Therefore βint(f(int(A))) = f(int(A)) $\subseteq$ f(A). Now f(int(A)) = βint(f(int(A))) $\subseteq$ βint(f(A))
(ii) ⇒ (iii) Let B be any IFS in Y. Then f^{-1}(B) is an IFS in X. (ii) implies that f(int(f^{-1}(B))) $\subseteq$ βint(f(f^{-1}(B))) = βint(B). Now int(f^{-1}(B)) $\subseteq$ f^{-1}(f(int(f^{-1}(B)))) $\subseteq$ f^{-1}(βint(B))
(iii) ⇒ (i) Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. (iii) implies that int(f^{-1}(f(A))) $\subseteq$ f^{-1}(βint(f(A))). Now A = int(A) $\subseteq$ int(f^{-1}(f(A))) $\subseteq$ f^{-1}(βint(f(A))). Therefore f(A) $\subseteq$ f(f^{-1}(βint(f(A)))) = βint(f(A)) $\subseteq$ f(A). This implies βint(f(A)) = f(A). Hence f(A) is an IFβOS in Y. Since every IFβOS is an IFGβOS, f(A) is an IFGβOS in Y. Thus f is an IFGβOM.

Theorem 3.22: A mapping f: X → Y is an IFGβOM if and only if int(f^{-1}(B)) $\subseteq$ f^{-1}(βint(B)) for every B $\subseteq$ Y, where Y is an IFβ$T_{1/2}$ space.

Proof: Necessity: Let B $\subseteq$ Y. Then f^{-1}(B) $\subseteq$ X and int(f^{-1}(B)) is an IFOS in X. By hypothesis, f(int(f^{-1}(B))) is an IFGβOS in Y. Since Y is an IFβ$T_{1/2}$ space, f(int(f^{-1}(B))) is an IFβOS in Y. Therefore f(int(f^{-1}(B))) = βint(f(int(f^{-1}(B)))) $\subseteq$ βint(B). This implies int(f^{-1}(B)) $\subseteq$ f^{-1}(βint(B)).

Sufficiency: Let A be an IFOS in X. Therefore int(A) = A. Then f(A) $\subseteq$ Y. By hypothesis int(f^{-1}(f(A))) $\subseteq$ f^{-1}(βint(f(A))). That is int(A) $\subseteq$ int(f^{-1}(f(A))) $\subseteq$ f^{-1}(βint(f(A))). Therefore A $\subseteq$ f^{-1}(βint(f(A))). This implies f(A) $\subseteq$ βint(f(A)) $\subseteq$ f(A). Hence f(A) is an IFβOS in Y and hence an IFGβOS in Y. Thus f is an IFGβOM.

Theorem 3.23: Let f: X → Y be an onto mapping where Y is an IFβ$T_{1/2}$ space. Then f is an IFGβOM if and only if for any IFP c(α, β) ∈ Y and for any IFN B of f^{-1}(c(α, β)), there is an IFβN A of c(α, β) such that c(α, β)∈ A and f^{-1}(A) $\subseteq$ B.

Proof: Necessity: Let c(α, β) ∈ Y and let B be an IFN of f^{-1}(c(α, β)). Then there is an IFOS C in X such that f^{-1}(c(α, β)) ∈ C $\subseteq$ B. Since f is an IFGβCM, f(C) is an IFGβOS in Y. Since Y is an IFβ$T_{1/2}$ space, f(C) is an IFβOS in Y and c(α, β) ∈ f(f^{-1}(c(α, β))) $\subseteq$ f(C) $\subseteq$ f(B). Put A = f(C). Then A is an IFβN of c(α, β) and c(α, β) ∈ A $\subseteq$ f(B). Thus c(α, β) ∈ A and f^{-1}(A) $\subseteq$ f^{-1}(f(B)) = B. That is f^{-1}(A) $\subseteq$ B.

Sufficiency: Let B $\subseteq$ X be an IFOS. If f(B) = $0_\sim$ then there is nothing to prove. Suppose that c(α, β) ∈ f(B). This implies f^{-1}(c(α, β)) ∈ B. Then B is an IFN of f^{-1}(c(α, β)). By hypothesis there is an IFβN A of c(α, β) such that c(α, β) ∈ A and f^{-1}(A) $\subseteq$ B. Therefore there is an IFβOS C in Y such that c(α, β) ∈ C $\subseteq$ A = f(f^{-1}(A)) $\subseteq$ f(B). Hence f(B) = ∪ { c(α, β) / c(α, β) ∈ f(B} $\subseteq$ ∪ {$C_{c(\alpha,\beta)}$ / c(α, β) ∈ f(B) } $\subseteq$ f(B). Thus f(B) = ∪ { $C_{c(\alpha,\beta)}$ / c(α, β) ∈ f(B) }. Since each C is an IFβOS, f (B) is also an IFβOS and hence is an IFGβOS in Y. Therefore f is an IFGβOM.

Theorem 3.24: Let f: X → Y be a bijective mapping, where X is an IFβ$T_{1/2}$ space. Then the following are equivalent:
(i) f is an IFMGβCM
(ii) for each IFP c(α, β) ∈ Y and every IFβN A of f^{-1}(c(α, β)), there exists an IFGβOS B in Y such that c(α, β) ∈ B $\subseteq$ f(A).
(iii) for each IFP c(α, β) ∈ Y and every IFSN A of f^{-1}(c(α, β)), there exists an IFGβOS B in Y such that
c(α, β) ∈ B and f^{-1}(B) $\subseteq$ A.

Proof: (i) ⇒ (ii) Let c(α, β) ∈ Y and A the IFSN of f^{-1}(c(α, β)). Then there exists an IFβOS C in X such that f^{-1}(c(α, β)) ∈ C $\subseteq$ A. Since every IFβOS is an IFGβOS, C is an IFGβOS in X. Then by hypothesis, f(C) is an IFGβOS in Y. Now c(α, β) ∈ f(C) $\subseteq$ f(A). Put B = f(C). This implies c(α, β) ∈ B $\subseteq$ f(A).
(ii) ⇒ (iii) Let c(α, β) ∈ Y and A the IFSN of f^{-1}(c(α, β)). Then there exists an IFβOS C in X such that f^{-1}(c(α, β)) ∈ C $\subseteq$ A. Since every IFβOS is an IFGβOS, C is an IFGβOS in X. Then by hypothesis, f(C) is an IFGβOS in Y.
Now c(α, β) ∈ f(C) $\subseteq$ f(A). Put B = f(C). This implies c(α, β) ∈ B $\subseteq$ f(A). Now f^{-1}(B) $\subseteq$ f^{-1}(f(A)) $\subseteq$ A. That is f^{-1}(B) $\subseteq$ A.
(iii) ⇒ (i) Let A be an IFGβOS in X. Since X is an IFβ$T_{1/2}$ space, A is an IFβOS in X. Let c(α, β) ∈ Y and f^{-1}(c(α, β)) ∈ A. That is c(α, β) ∈ f(A). This implies A is an IFβN of f^{-1}(c(α, β)). Then by hypothesis, there exists an IFGβOS B in Y such that c(α, β) ∈ B and f^{-1}(B) $\subseteq$ A. Hence by Theorem 2.18, f(A) is an IFGβOS in Y. Therefore f is an IFMGβCM.

Theorem 3.25: If f: X → Y be a bijective mapping then the following are equivalent:

(i) f is an IFMGβCM

(ii) f(A) is an IFGβOS in Y for every IFGβOS A in X

(iii) for every IFP c(α, β) ∈ Y and for every IFGβOS B in X such that f^{-1}(c(α, β)) ∈ B, there exists an IFGβOS A in Y such that c(α, β) ∈ A and f^{-1}(A) ⊆ B.

Proof: (i) ⇒ (ii) is obvious, since f is bijective.

(ii) ⇒ (iii) Let c(α, β) ∈ Y and let B be an IFGβOS in X such that f^{-1}(c(α, β)) ∈ B. This implies c(α, β) ∈ f(B). By hypothesis, f (B) is an IFGβOS in Y. Let A = f (B). Therefore c(α, β) ∈ f(B) = A and f^{-1}(A) = f^{-1}(f(B)) ⊆ B.

(iii) ⇒ (i) Let B be an IFGβCS in X. Then B^c is an IFGβOS in X. Let c(α, β) ∈ Y and f^{-1}(c(α, β)) ∈ B^c. This implies c(α, β) ∈ f(B^c). By hypothesis there exists an IFGβOS A in Y such that c(α, β) ∈ A and f^{-1}(A) ⊆ B^c. Put A = f(B^c). Then c(α, β) ∈ f(B^c) and A = f(f^{-1}(B^c)) ⊆ f(B^c). Hence by Theorem 2.18, f(B^c) is an IFGβOS in Y. Therefore f(B) is an IFGβCS in Y. Thus f is an IFMGβCM.

Theorem 3.26: If f: X → Y be a bijective mapping then the following are equivalent:

(i) f is an IFMGβCM

(ii) f(A) is an IFGβOS in Y for every IFGβOS A in X

(iii) f(βint(B)) ⊆ βint(f(B)) for every IFS B in X

(iv) βcl(f(B)) ⊆ f(βcl(B)) for every IFS B in X.

Proof: (i) ⇒ (ii) is obvious.

(ii) ⇒ (iii) Let B be any IFS in X. Since βint(B) is an IFβOS, it is an IFGβOS in X. Then by hypothesis, f(βint(B)) is an IFGβOS in Y. Since Y is an IFβ$T_{1/2}$ space, f(βint(B)) is an IFβOS in Y. Therefore f(βint(B)) = βint(f(βint(B))) ⊆ βint(f(B)).

(iii) ⇒ (iv) can easily proved by taking complement in (iii).

(iv) ⇒ (i) Let A be an IFGβCS in X. By hypothesis, βcl(f(A)) ⊆ f(βcl(A)). Since X is an IFβ$T_{1/2}$ space, A is an IFβCS in X. Therefore βcl(f(A)) ⊆ f(βcl(A)) = f(A) ⊆ βcl(f(A)). Hence f(A) is an IFβCS in Y and hence an IFGβCS in Y. Thus f is an IFMGβCM.

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